XII. *Account of Experiments on Torsion and Flexure for the Determination of Rigidities.* —Third Paper. *By* JOSEPH DAVID EVERETT, *D.C.L., Professor of Natural Philosophy in Queen's College, Belfast. Communicated by Sir* WILLIAM THOMSON, *F.R.S.*

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THESE experiments are a continuation of those described in my papers read February 26th, 1866 and February 7th, 1867. They were made during the Winter Session of 1866–67, in the same place, and with the same apparatus as the later experiments on the steel rod (see p. 150 of last paper), the observer being Mr. ZACCHEUS WALKER, student of Engineering, who has been already mentioned as joint observer in the experiments on the steel rod. I personally inspected the apparatus from time to time, and assisted in the taking of those measurements which only required to be made once for all in the case of each rod. The weights, in air and water, of the portions of rods operated on, were observed by Mr. JOHN TATLOCK, Laboratory Assistant to Sir WILLIAM THOMSON, who also performed this portion of the labour in the previous experiments.

The rods operated on in the experiments now to be described were three in number, and were of malleable iron, cast iron, and copper respectively, those previously operated on being of flint glass (two specimens), brass, and steel.

Experiments on Malleable Iron Rod.

The mean values of T (proportional to torsion) and F (proportional to flexure) were as follows, the pointer-readings indicating the amounts by which the rod had been rotated about its axis from a certain initial position.

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
30°	210·3	165·3	120°	210·7	165·0
60°	212·0	164·8	150°	210·1	165·0
90°	210·3	165·3	180°	210·8	165·2

Combining, as theory requires, those positions which are mutually at right angles, we have—

From 30° and 120° . . .	T=210·5	F=165·2
„ 60° „ 150° . . .	„ 211·0	„ 164·9
„ 90° „ 180° . . .	„ 210·6	„ 165·2

The cross-piece on whose arms the weights are hung, ought to have been turned through a right angle in passing from the position 90° to the position 120°, in order to

destroy the effect of inequality in the length of the arms. This operation was, however, omitted through oversight, and it is therefore necessary, in comparing the torsional and flexural rigidities, to correct for this difference.

The arm of couple, that is the distance SS′ or TT′ in fig. 1 of last paper, was 55·71 centims. for torsion and 55·83 for flexure.

Also the units in which the numbers T and F are expressed, being tenths of scale-divisions perpendicular and parallel to the rod respectively, are slightly different, being $\frac{23\cdot88}{1710}$ and $\frac{23\cdot97}{1710}$ centims. for torsion and flexure respectively.

The corrections for these two inequalities are in opposite directions. Their resultant is a correction of $\cdot0016\,\frac{T}{F}$ to be subtracted.

The values of $\frac{T}{F}$ thus corrected are—

From 30° and 120°	. . .	1·272,
„ 60° „ 150°	. . .	1·278,
„ 90° „ 180°	. . .	1·273,

which when diminished by unity are determinations of σ or POISSON'S ratio, subject to two other corrections, which we now proceed to calculate.

For the mechanical correction the data are—

Torsion in portion of rod between mirrors	. . .	·00661,
Flexure in portion of rod between mirrors	. . .	·00519,

these numbers (which are in circular measure) being obtained by dividing T and F respectively reduced to centimetres by 445·6, which is twice the height of scale above mirrors. They must now be multiplied by $\frac{452}{294}$, which is the ratio of all that portion of the rod subjected to torsion and flexure to the portion between mirrors, and also by the constant number ·729 (see last paper, p. 143). The products thus obtained are ·0074 and ·0058; hence the mechanical corrections of T and F are

$$+\cdot0074\ \mathrm{T}\ \text{and}\ +\cdot0058\ \mathrm{F}.$$

The optical correction (which I am told was too briefly described in my last paper) is dependent on the fact that the ray from scale to mirror is not precisely vertical, and therefore not truly perpendicular to the scale, which is horizontal. It is determined by measuring—

(1) Height of scale above mirrors.

(2) Distance of vertical through centre of scale from line joining centres of mirrors.

(3) Distances, resolved parallel to rod, of vertical through centre of scale from centre of each mirror.

These three distances suffice to determine the corrections both for torsion and flexure, on the hypothesis that the centre of the scale comes into the field of view of both telescopes in every observation, an hypothesis which, though not strictly fulfilled, gives a fair approximation to the actual obliquities.

Let β (as in last paper) denote the quotient of one of the distances (2) or (3) according as we are dealing with torsion or flexure, by the distance (1). Then since β is the tangent of a small angle (which call B), we have

$$\delta \tan B = (1+\tan^2 B)\delta B = (1+\beta^2)\delta B,$$

and therefore the angle turned by mirror (δB) is found by dividing the observed deflection ($\delta \tan B$) by $1+\beta^2$, or, what amounts to the same thing, multiplying it by $1-\beta^2$.

Let B_1, β_1 and B_2, β_2 be the values of B, β for the nearer and further mirrors respectively; then $\delta \tan B_2 - \delta \tan B_1 = (1+\beta_2^2)\delta B_2 - (1+\beta_1^2)\delta B_1$, which may be shown to be equal to

$$(\delta B_2 - \delta B_1)\left(1+\frac{m_2\beta_2^2 - m_1\beta_1^2}{m_2 - m_1}\right),$$

m_2 and m_1 being any two numbers which have the ratio $\delta \tan B_2 : \delta \tan B_1$. The observed values of T and F are proportional to $\delta \tan B_2 - \delta \tan B_1$, and must therefore be multiplied by the correcting factor $1-\frac{m_2\beta_2^2 - m_1\beta_1^2}{m_2 - m_1}$, which, when β_1 and β_2 are equal, becomes simply $1-\beta^2$.

In the present series of experiments β_1 and β_2 were sensibly equal, their common value being about ·069 for torsion and ·064 for flexure. By squaring these numbers we obtain the corrections

$$-{\cdot}0047\ \mathrm{T} \quad \text{and} \quad -{\cdot}0041\ \mathrm{F},$$

which are applicable not only to the experiments on the malleable iron rod, but also to those on the cast-iron and copper rods.

Adding together the mechanical and optical corrections, we obtain the total corrections

$$+{\cdot}0027\ \mathrm{T} \quad \text{and} \quad +{\cdot}0017\ \mathrm{F}.$$

The resulting correction for $\frac{T}{F}$ is $+{\cdot}001\frac{T}{F}$, which applied to the values 1·272, 1·278, 1·273 gives, after subtracting unity, ·273, ·279, ·274 as the corrected determinations of the value of σ, and we adopt the mean of these, which is ·275.

We proceed to calculate the torsional and flexural rigidities t and f.

The mean values of T and F, uncorrected, are respectively 210·7 and 165·1, which, when corrected as above, become 211·3 and 165·4.

Now we have

$$t = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{1710}{23{\cdot}88} \div \mathrm{T},$$
$$f = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{1710}{23{\cdot}97} \div \mathrm{F},$$

where, in centimetres and grammes weight, we have twice distance=445·6, force=100, length=29·45, arm=55·71 for torsion and 55·83 for flexure.

Hence

$$\log t = 9{\cdot}71893 - \log \mathrm{T} - 7{\cdot}39403,$$
$$\log f = 9{\cdot}71823 - \log \mathrm{F} = 7{\cdot}49969.$$

The rod having been cut at the places where the mirrors were attached, the central

piece was weighed in air and water, the weights found being 100·485 and 87·400 grammes. The temperature of the water was 10·4 Cent., at which temperature the expansion of water is 1·0003. From these data the volume of the rod in centimetres is found to be

$$13{\cdot}085\times 1{\cdot}0003=13{\cdot}089,$$

and the specific gravity 100·485 ÷ 13·089 = 7·6771. Dividing the volume by the length, which was 29·27, we find the sectional area $\pi r^2=\cdot 44566$, $r=\cdot 37664$. Hence we have

$$\text{M (Young's modulus)}=\frac{4f}{\pi r^4}=1{,}999{,}400{,}000,$$

$$n\ \text{(Rigidity)}=\frac{2t}{\pi r^4}=783{,}790{,}000,$$

$$k\ \text{(Incompressibility)}=\frac{\mathrm{M}n}{3(3n-\mathrm{M})}=1{,}484{,}100{,}000,$$

$$\sigma\ \text{(Poisson's ratio)}=\frac{\mathrm{M}}{2n}-1=\cdot 275 \text{ as above.}$$

Experiments on Cast-Iron Rod.

These experiments gave the following values for T and F:—

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
0°	311·2	245·8	90°	311·8	245·2
30°	310·7	245·3	120°	310·8	245·0
60°	311·8	244·7	150°	311·7	245·3

Combining positions at right angles, we have—

From 0° and 90° . .	T=311·5	F=245·5
„ 30° „ 120° . .	„ 310·8	„ 245·2
„ 60° „ 150° . .	„ 311·7	„ 245·0

Here no correction is required for difference of arms, as the cross-piece was turned through a right angle in passing from the third to the fourth position.

The values of $\frac{\mathrm{T}}{\mathrm{F}}$, corrected for difference of scale-divisions by subtracting one part in 266, are 1·264, 1·263, and 1·267, which are first approximations to $1+\sigma$.

To find the mechanical correction, we have length between mirrors 28·69, length subjected to torsion and flexure 43·5, height of scale above mirrors 222·8, as before. From these data, together with the above values of T and F, we have—

Torsion in portion of rod between mirrors	·0098
Flexure „ „ „ „	·0077

which numbers being multiplied by $\frac{435}{287}\times\cdot 729$, give ·0108 and ·0085. Hence the mechanical corrections are

$$+\cdot 0108\,\mathrm{T} \text{ and } +\cdot 0085\,\mathrm{F};$$

and the optical corrections being the same as before, the total corrections are

$$+\cdot 0061\,\mathrm{T} \text{ and } +\cdot 0044\,\mathrm{F}.$$

The resulting correction for $\frac{T}{F}$ is $+{\cdot}0017\ \frac{T}{F}$, the application of which gives as the corrected determinations of σ, ·266, ·265, and ·269, and we adopt the mean ·267.

The mean uncorrected values of T and F are 311·3 and 245·2, which, when corrected as above, become 313·2 and 246·3.

For the torsional and flexural rigidities, we have

$$t=445{\cdot}6\times100\times55{\cdot}77\times28{\cdot}69\times\tfrac{1710}{23{\cdot}88}\div\mathrm{T},$$

$$f=445{\cdot}6\times100\times55{\cdot}77\times28{\cdot}69\times\tfrac{1710}{23{\cdot}97}\div\mathrm{F}.$$

Hence

$$\log t=9{\cdot}70805-\log\mathrm{T}=7{\cdot}21223,$$

$$\log f=9{\cdot}70641-\log\mathrm{F}=7{\cdot}31495.$$

The rod having been cut at the places where the mirrors were attached, the middle piece was found to weigh 90·110 in air, and 77·658 in water at temperature 9°·9 C. The expansion of water at this temperature being 1·00026, we have for the volume of the piece

$$12{\cdot}452\times1{\cdot}00026=12{\cdot}455,$$

and for the specific gravity,

$$90{\cdot}110\div12{\cdot}455=7{\cdot}2347.$$

The volume divided by the length, which was 28·66, gives $\pi r^2={\cdot}43458$, $r={\cdot}37193$.

Hence we have

$$\mathrm{M}=1{,}374{,}100{,}000,$$

$$n=542{,}340{,}000,$$

$$k=982{,}180{,}000,$$

$$\sigma={\cdot}267.$$

Experiments on Copper Rod.

These experiments gave the following values for T and F:—

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
0°	365·0	265·0	90°	365·8	264·8
30°	365·2	265·0	120°	364·7	265·0
60°	365·8	264·7	150°	365·7	265·2

Combining positions at right angles, we have—

From 0° and 90° . .	T=365·4	F=264·9
„ 30° „ 120° . .	„ 365·0	„ 265·0
„ 60° „ 150° . .	„ 365·7	„ 264·9

The values of $\frac{T}{F}$ corrected for difference of scale-divisions, are 1·374, 1·372, and 1·375.

To find the mechanical correction, we have length between mirrors 28·72, length subject to torsion and flexure 45·5, height of scale 222·8.

Hence we find

Torsion in portion of rod between mirrors . . .	·0114,
Flexure " " " "	·0083,

which numbers being multiplied by $\frac{455}{287} \times \cdot 729$ give ·0132 and ·0096. Hence the mechanical corrections are

$$+\cdot 0132\,\mathrm{T} \text{ and } +\cdot 0096\,\mathrm{F},$$

which being added to the optical corrections already specified, give the total corections

$$+\cdot 0085\,\mathrm{T} \text{ and } +\cdot 0055\,\mathrm{F},$$

and a resultant correction $+\cdot 003\frac{\mathrm{T}}{\mathrm{F}}$. The application of this correction gives as the corrected determinations of σ, ·378, ·376, and ·379, and we adopt the mean ·378.

The corrected values of T and F are 368·5 and 266·4.

For the torsional and flexural rigidities, we have

$$t = 445\cdot 6 \times 100 \times 55\cdot 77 \times 28\cdot 72 \times \tfrac{1710}{23\cdot 88} \div \mathrm{T},$$

$$f = 445\cdot 6 \times 100 \times 55\cdot 77 \times 28\cdot 72 \times \tfrac{1710}{23\cdot 97} \div \mathrm{F}.$$

Hence

$$\log t = 9\cdot 70850 - \log \mathrm{T} = 7\cdot 14206,$$

$$\log f = 9\cdot 70686 - \log \mathrm{F} = 7\cdot 28133.$$

The rod having been cut at the places where the mirrors were attached, the middle piece was found to weigh 110·645 in air, and 98·136 in water at temperature 10° C. The expansion of water at this temperature being 1·00026, we have for the volume of the piece

$$12\cdot 502 \times 1\cdot 00026 = 12\cdot 5122;$$

and for its specific gravity,

$$110\cdot 645 \div 12\cdot 5122 = 8\cdot 84293.$$

The volume divided by the length, which was 28·61, gives $\pi r^2 = \cdot 43733$, $r = \cdot 37310$.

Hence

$$\mathrm{M} = 1{,}255{,}800{,}000,$$

$$n = 455{,}640{,}000,$$

$$k = 1{,}716{,}400{,}000,$$

$$\sigma = \cdot 378,$$

the units of length and force being the centimetre and gramme weight.

The values of n obtained by Sir W. THOMSON for brass and copper from observations on the torsional vibrations of wires* were, in millions of grammes weight per square centimetre,

Brass, three specimens	410·3	354·8	350·1
Copper, two specimens	448·7	448·4.	

Other specimens of copper in abnormal states gave results ranging from 393·4 to 472·9.

* Proceedings, Royal Society, May 18, 1865.

The following Table contains the collected results of all my experiments, the figures I., II., III. indicating the paper in which the experiments are described and the results deduced. The values of M, n, and k are given in millions of grammes weight per square centimetre.

	M (Young's modulus).	n (Rigidity).	k (Resistance to compression).	σ (Poisson's ratio).	Specific gravity.
Glass, flint (I.)	614·3	244·2	423·0	·258	2·942
" " (II.)	585·1	239·0	353·3	·229	2·935
Brass, drawn (II.) ..	1094·8	372·9	5701 (?)	·469	8·471
Steel (II.)	2179·3	834·1	1875·6	·310	7·849
Iron, wrought (III.)	1999·4	783·8	1484·1	·275	7·677
" cast (III.)	1374·1	542·3	982·2	·267	7·235
Copper (III.)	1255·8	455·6	1716·4	·378	8·843

The values of M and n are derived directly from the observed amounts of flexure and torsion respectively, and their probable errors (estimated as percentages) may be considered equal.

Again, the comparison between flexure and torsion is so direct (with the exception of the first set of results for glass), that the probable error of $\frac{M}{2n}$ may be considered about equal to that of M or n.

The values of σ and k are deduced on the hypothesis that the substances operated on are isotropic; and on this hypothesis, if e denote the probable error of $\frac{M}{2n}$, the probable error of σ is $\frac{1+\sigma}{\sigma}e$, and that of k is $\frac{1+\sigma}{\frac{1}{2}-\sigma}e$ nearly.

For all the substances in the Table except brass, the value of the coefficient $\frac{1+\sigma}{\frac{1}{2}-\sigma}$ is from 5 to 11, but for brass it is 47; hence the enormous value of k found for brass cannot be depended on.

The tendency of anything like fibrous structure in the rods operated on, the fibres being supposed to run in the direction of the length, is to make σ and k come out too large. This follows from the theoretical considerations adduced on page 152 of my last paper, and I have verified it by experiments on a rod of wood which gave values of T four or five times greater than those of F, whereas the ratio $\frac{T}{F}$ for isotropic substances must always lie between 1 and 1·5.

As the intention was expressed in my last paper of introducing a modification of the apparatus with the view of diminishing the mechanical correction, I should state by way of explanation that the experiments here described were nearly completed before that intention was expressed.